Modeling and Analysis of Interconnected Systems in the Presence of Uncertainty

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How should we simulate the

Global Financial System?

Ideally, what should be the

"Architecture of the Financial System?"

Modeling of Interconnected Systems

Dynamical View: Space of Random Trajectories

Not Equilibrium View

Tearing $\Longrightarrow$ Open Sub-systems;

Modeled to extent possible

Theory of Interconnection of Uncertain Systems

Composition of Stochastic Kernels to create Joint Probability distribution

Probability Theory allows us to compose "Heterogeneous" stochastic systems

Special kind of interconnection: feedback through a stochastic dynamical system

Stability of Interconnected System is obtained through "lossless" interconnection of dissipative systems

Feedback Interconnection of Economic System and Financial System

For example, we could study the stability of a Complex Power System using the methodology.

Part of Modeling: Parameterization and Identification of Parameters from Data

Measurement and data generation

Model Uncertainty and Robustness

Example:

Recent work in Finance to deal with Model Uncertainty (Knightian Uncertainty)

Also, Levy Processes replacing Brownian Motion

De Finetti's ideas of Coherence

Lessons from Power Systems

In Cascading Power Transmission, component outages propagate non-locally

Using data from many Cascading Failure Simulations, one can construct a Markovian Influence Graph (which is not the Grid topology) where Cascading Power Outages propagate locally

Model Reduction

Blackout data suggest that the frequency of large blackouts is governed by a power law

Power law makes the risk of large blackouts consequential, and is consistent with the Power System operating at a critical point due to steady increase in loading, maximum use of the grid and other factors

Real-Time Pricing in Electrical Power Systems may cause Instability due to Feedback

Section 1

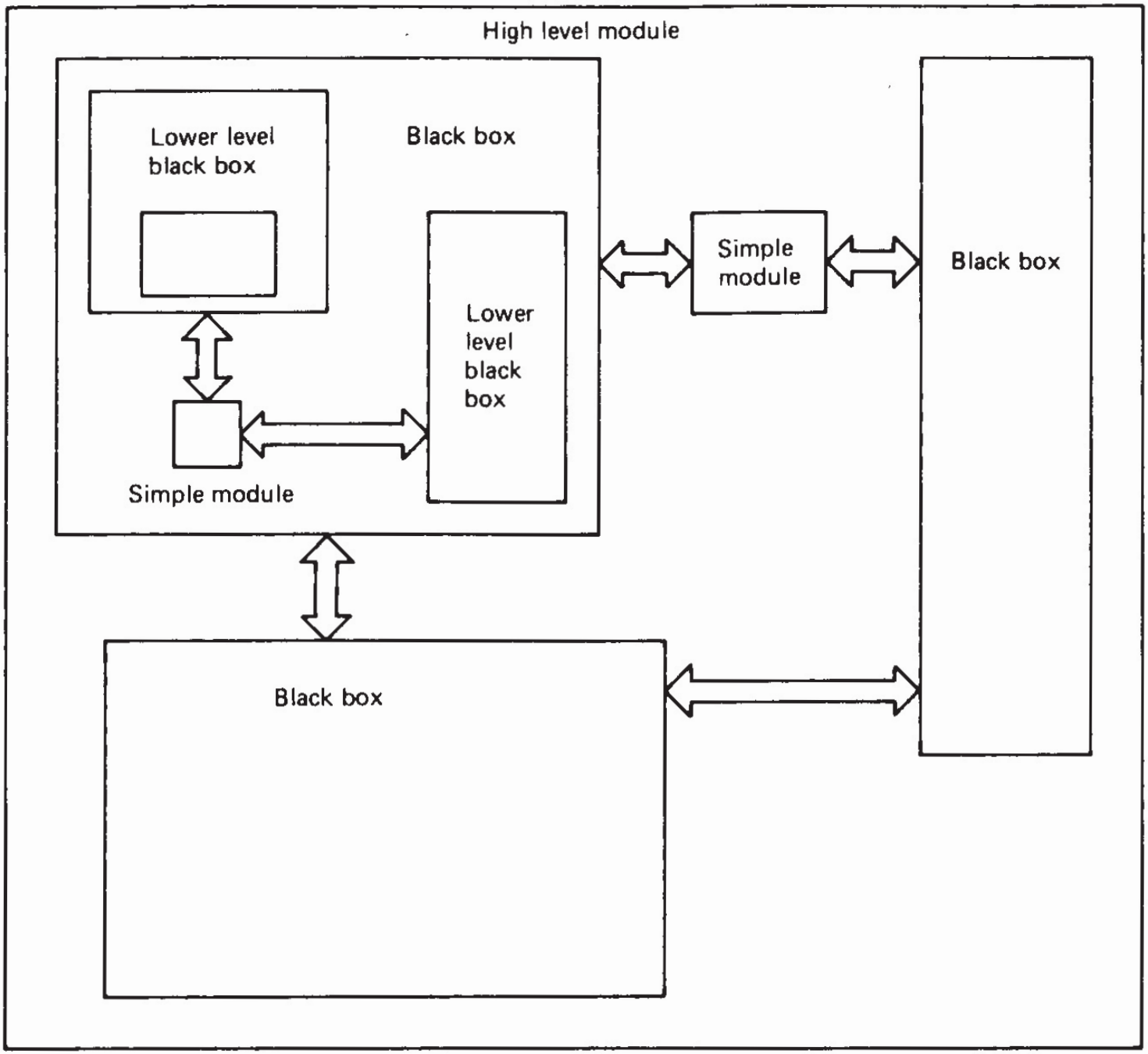
Architecture: Definition

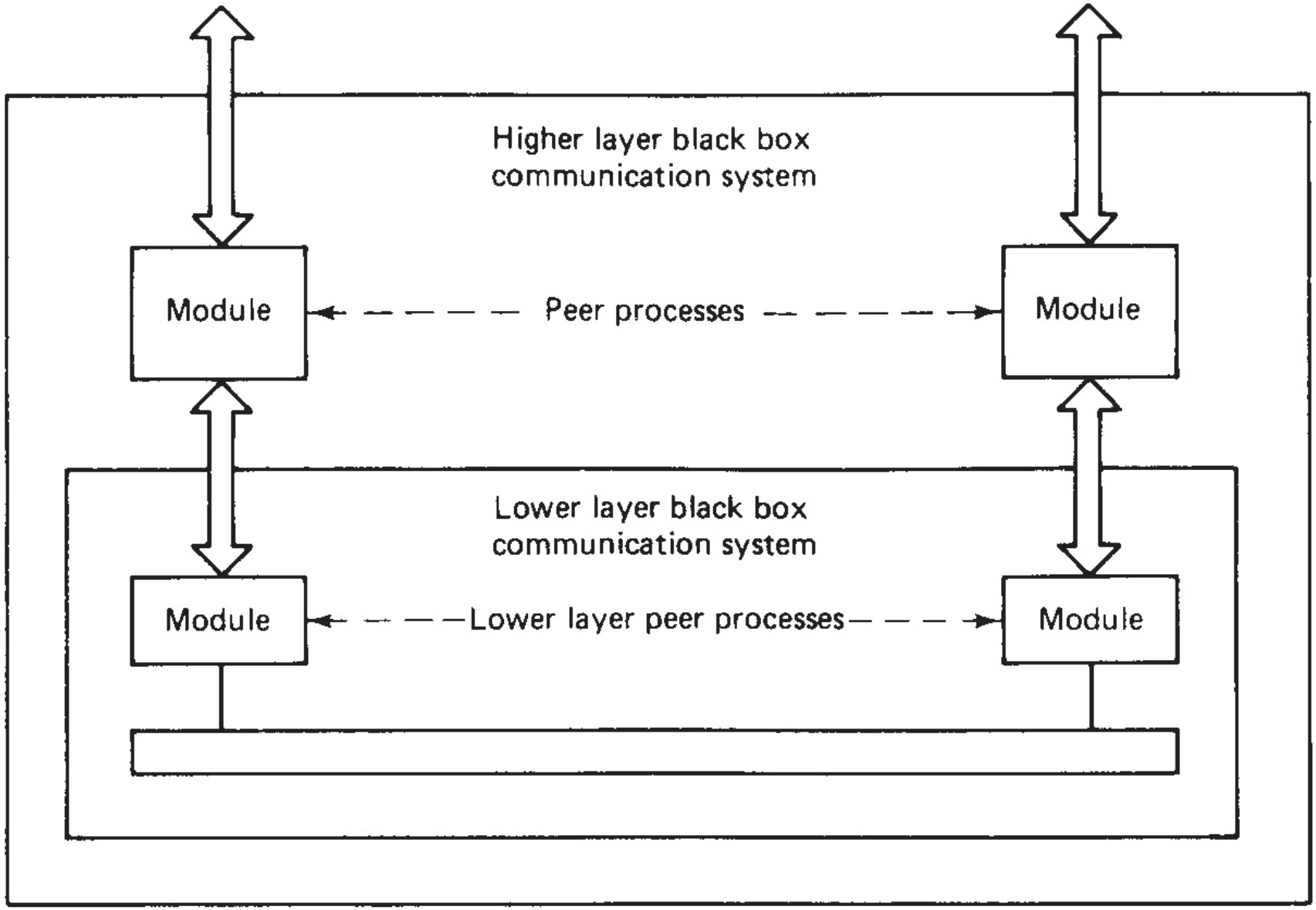
Architecture is the organization of Distributed Algorithms in hardware and software for performing complex heterogeneous functions.

Section 2

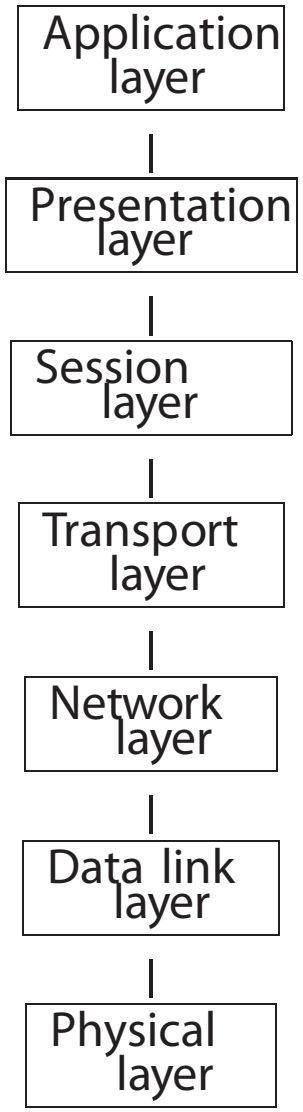
Architecture of the Internet

"Layering" the Financial System


High level module
Lower level black box
Black box
Lower level black box
Simple module
Simple module
Black box
Black box


Higher layer black box
communication system
Module
Peer processes
Module
Lower layer black box
communication system
Module
Lower layer peer processes
Module

Layered Architecture



Applications are run here

Encryption, compression, conversion

Allows for establishing connections between end hosts

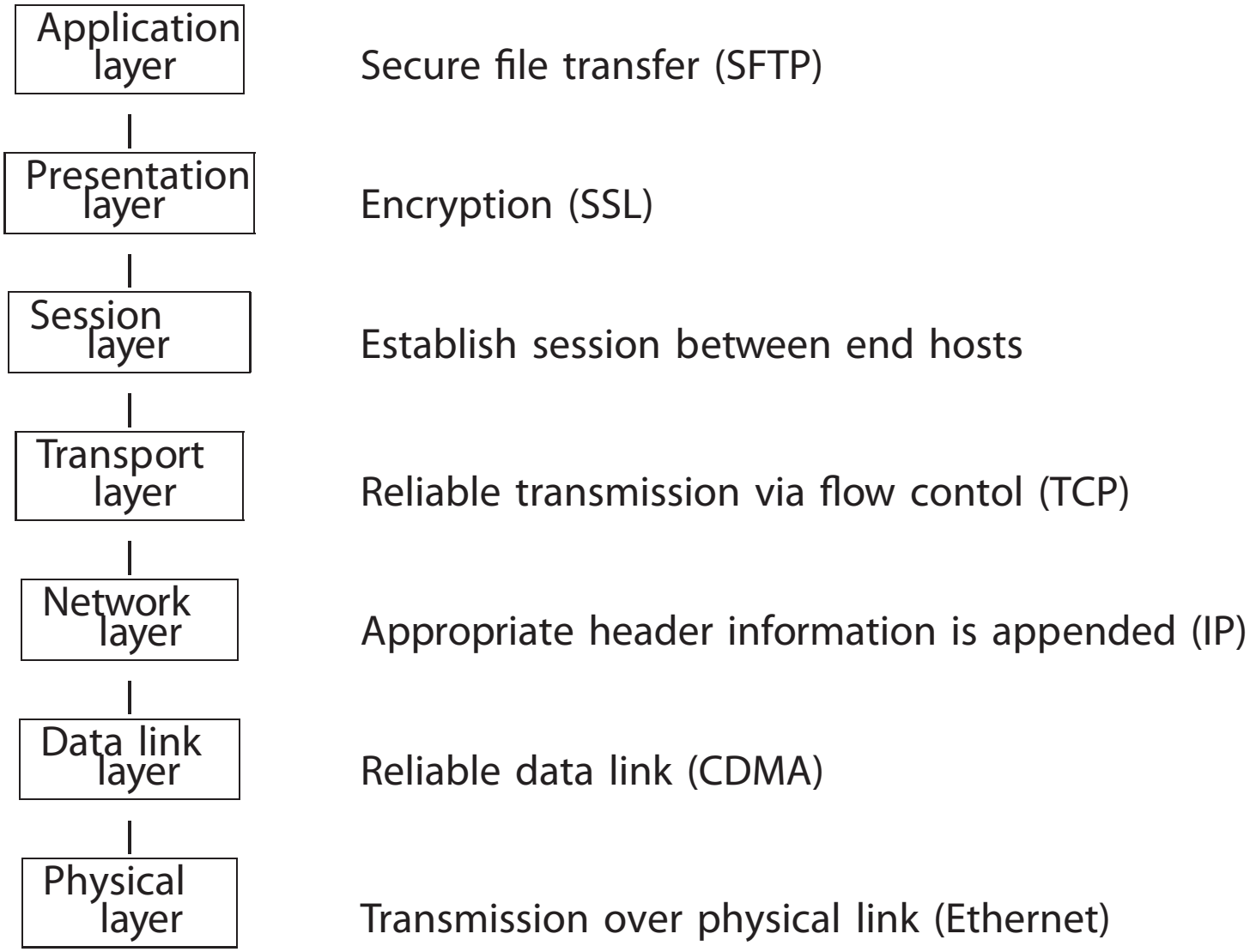
Transporting data between end-hosts and possibly converting unreliable network layer into reliable transport

Determines routing of data packets, appends necessary information before giving to data layer

Convert unreliable physical layer into reliable higher level data link

Provide link for bit transmission

Layered Architecture: An Example



References

(1) Two papers

- ``The Design Philosophy of The DARPA Internet Protocols,'' by D. Clark
- ``Architectural Considerations for a New Generation of Protocols,'' by D. Clark and D. L. Tennenhouse

(2) Data Networks, by Bertsekas–Gallager

- Chapter 1.3

(3) Expository article

- MIT Tech Review, Dec 05/Jan 06 issue, article, p. 62–69

Features of Layered Architectures for Performing Heterogeneous Functions

→ Structure of Layers: by analogy.

Two adjacent layers are "conditionally independent" give the interface linking the two layers

→ Interconnection view

→ Partial order on layers

→ Regulation in Financial systems ≡ Interfaces

$\rightarrow$ Higher order layers are "quotients" of lower order layers equipped with an equivalence relation.

For example, in Vision, where image is defined as a function $f : \mathcal{M} \subset \mathbb{R}^2 \rightarrow \mathbb{R}$, the equivalence relation is induced by a group action, the semi-direct, direct product of the Euclidian Group on $\mathrm{dom}(f) \subset \mathbb{R}^2$, appropriate group on $\mathrm{Range}(f)$ and the gauge group on the space of Images, resulting in a quotient

(cf: the work of Stefano Soatto)

$\rightarrow$ Quotient is often finite: Symbols

→ Layered Structure depends on functions that the system is required to perform and properties the system should possess (for example, robustness of performance against uncertainties).

→ "Autonomy of Layers" vs. Adaptation Possibilities

Section 3

What the Frog's Eye Tells the Frog's Brain

J.Y. Lettvin, H.R. Maturanat, W.S. McCulloch II, and W.H. Pitts

Research Laboratory for Electronics
MIT

Proc. Inst. of Radio Engr., 1959.

Conclusions

What are the consequences of this work? Fundamentally, it shows that the eye speaks to the brain in a language already highly organized and interpreted, instead of transmitting some more or less accurate copy of the distribution of light on the receptors. As a crude analogy, suppose that we have a man watching the clouds and reporting them to a weather station. If he is using a code, and one can see his portion of the sky too, then it is not difficult to find out what he is saying.

Methodological Advances

Statistical Inference and Optimization: High-dimensional Statistics

(Parameter space $\gg$ size of data)

Compressed Sensing

Manifold Learning: High-dimensional data often lies on or near low-dimensional manifold

Integration of Heterogeneous Data Sets

Structure in Data: Number of connected components, homology groups

Methodological Advances (continued)

Data Assimilation: Bayesian Inference

Equivalent to Free-Energy Minimization

Pattern Recognition (Empirical Risk Minimization)

Clustering

Data to Models

Network Vulnerability and Security

Systemic Risk

MicroEconomics vs. MacroEconomics

Need:

Sufficiently rich behavioral modeling of heterogeneous interacting economic agents of Micro level, leading to emergent phenomena at Macro level

Gibbs Measures and Phase Transitions

Modeling of Human Behavior

Linquistic Description: Minimalist View of Noam Chomsky