Modeling Investment Behavior and Risk Propagation in Financial Networks using Inverse Optimization *

John R. Birge

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Abstract

Connections among institutions in the global financial network create the potential for risk to propagate and for failures to cascade as successive institutions fail. As conditions, such as capital requirements change, institutions may modify their behavior in ways that can fundamentally change the relationships among institutions and lead to substantially different failure dynamics. Increasing capital requirements can, for example, paradoxically increase the potential for failures to propagate by altering the intensity of relationships and risk exposures. Predicting such outcomes and directing policies to reduce overall systemic risk requires modeling of institutional responses to environmental conditions. This paper discusses an approach based on inverse optimization of relationship decisions subject to capital constraints. A model of cascading failures and data from national debt cross-holdings illustrate the approach and demonstrate how changing capital requirements may lead to distinct differences in the sequences and extent of failures. **Keywords:** systematic risk, structural estimation, capital requirements

1 Introduction

The interconnected relationships among global financial institutions creates a complex network that can transmit risk in indirect ways across the network, creating a systemic form of risk associated with the structure of the network. Regulatory capital requirements create buffers against shocks

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in this network that can reduce the potential for cascading failures that build sequentially as the values of individual institutions fall due to their cross-holdings in the previously failed entities. As noted in Acharya (2009) and elsewhere, however, regulatory measures should consider the institutions' endogenous response to requirements, which may yield increasingly correlated investments that exacerbate the potential for systemic failures. How institutions respond to regulation, however, may depend on individual and bilateral benefit and cost perceptions that are not directly observable. This paper considers an approach to model the consequences for the network equilibrium of such variations and a procedure to estimate the effects of environmental changes. The results are illustrated using cross-holdings in the developed economies' banking network.

The role of financial intermediation, the structure of these firms, and the impact of regulation through such mechanisms as mandatory deposit insurance and regulatory capital requirements has been extensively studied both theoretically and empirically (see, e.g., the survey in Gorton and Winton (2003)). Within this vast literature, many papers have highlighted the potential negative impact of tighter capital requirements. Kashyap et al. (2008), for example, focuses on the agency cost that increased equity entails in terms of higher monitoring cost or other management inefficiency. Others that explore the role of equity and capital requirements include Berger et al. (1995), Froot and Stein (1998), Diamond and Rajan (2000), Hellmann et al. (2000), Morrison and White (2005), and Allen et al. (2015). A common issue in these papers is the cost of equity and often the potential distortion of constraints. Another group of papers, including Allen and Gale (2000), Freixas et al. (2000), Eisenberg and Noe (2001), Dasgupta (2004), Cifuentes et al. (2005), Acharya et al. (2006), Gai (2010), Gauthier et al. (2012), Gourieroux et al. (2012), and Elliott et al. (2014) has focused on the interconnectedness of institutional holdings and investments and the potential for cascading failures. This papers fits into this stream of work with a focus on the implications of the endogeneity of inter-institutional connections and their dependence on exogenous regulatory requirements.

The basic building block for the approach is the network model in Elliott et al. (2014) with the addition of endogenous organizational decisions on cross-holdings, which may change dynamically in response to shocks across the network. The emphasis is on how this dynamic response can dramatically alter the propagation of shocks and lead to quite different failure cascades than those that occur when the organizations only react passively to failures as in previous models. The differences follow from the organizations' relative advantages in assessing inter-organizational investment opportunities and the effects of differing market rate dynamics (as, for example, explored empirically in Birge and Júdice (2013) and used to simulate individual failure events in Birge and Júdice (2014)). Simulations using data on cross-country debt holdings as in Elliott et al. (2014) demonstrate how tightening regulatory capital requirements can lead to instability through cross-holding responses that lead to greater exposures and a more fragile network.

The calibration of the model for observed cross-holding data follows a structural estimation or inverse optimization to identify the relative benefits and costs of cross-holdings assuming each organization maximizes expected utility given current rates and requirements. The optimality conditions for these subproblems then allow identification of the network (as also done for electricity networks in Birge et al. (2014)). This approach is similar to other uses of optimization conditions directly for structural estimation as in Su and Judd (2012) and references therein. The following section describes the basic network model that we follow. Section 3 presents the inverse optimization model that is used to estimate the relevant parameters. Section 4 then provides an illustrative numerical example using international cross-holding data and a counter-factual analysis of the effects of varying capital requirements. Section 5 then concludes the paper and suggests additional studies.

2 Network Model

The basic model here follows the form in Elliott et al. (2014) with the addition of endogenous investment choices that are assumed to determine the level of cross-holdings of the institutions that are represented as nodes. Each institution is then assumed to maximize a utility based on expected returns and risk in each other organization and additional costs that can be interpreted as transaction costs associated with the investments, such as monitoring costs or effective costs that might represent imbalances relative to assets or liabilities not represented in the model as given.

Formally, the model depicts the n institutions as distinct nodes, $i = 1, \dots, n$. Each institution chooses cross-holdings c_{ij} (forming the matrix C) that represents the fraction of institution j owned by i (where $c_{ij} \geq 0$, $c_{ii} = 0$, and $\sum_{i=1}^{n} c_{ij} \leq 1$). Following Elliott et al. (2014), $\hat{C}$ is the diagonal matrix of outside holdings (i.e., shares held outside of the network) formed by $\hat{c}_{ii} = 1 - \sum_{j=1}^{n} \hat{c}_{ji}$ and $\hat{c}_{ij} = 0$ for $j \neq i$. The model also includes m primitive assets with prices $p_k, k = 1, \ldots, m$ and ownership shares d_{ik} (matrix D) to represent the fraction of k owned by i.

These fundamentals lead to a linear equation for the vector w of total values of each institution as

$$w = Dp + Cw, \tag{2.1}$$

or, assuming $I - C$ is invertible,

$$w = (I - C)^{-1} Dp. \tag{2.2}$$

As in Elliott et al. (2014), we also identify a distinct market value v_i of each organization i to represent the value held by external investors so that $v = \hat{C}w$ or

$$v = \hat{C}(I - C)^{-1} Dp, \tag{2.3}$$

and follow Elliott et al. (2014) in using $A = \hat{C}(I - C)^{-1}$ so that $v = ADp$.

In Elliott et al. (2014), this basic network is used to explore potential cascades of failures that can result if the value v_i of any organization i falls below a critical value $\bar{v}_i$. Using their notation, the cost of such a failure is a fraction $0 < \beta_i < 1$ of the value v_i so that v_i is reduced by $b_i(v, p) = \beta_i(p) \mathbf{1}_{v_i < \bar{v}_i}$ where the dependence on p allows for the proportional losses given failures to depend on the primitive asset values. In Elliott et al. (2014), the equilibrium values must then satisfy

$$v = \hat{C}(I - C)^{-1}(Dp - b(p, v)), \tag{2.4}$$

which, as Elliott et al. (2014) observes, also has a solution but may have many solutions. Elliott et al. (2014) describe a simple algorithm iteratively updating $b(p, v)$ as a cascade of failures, which then finds the *best-case* equilibrium in which the fewest number of organizations fail.

The point of departure in this paper from Elliott et al. (2014) is that instead of assuming that C is exogenous, the model here assumes that C results from incentive-compatible choices of the institutions. In the illustration below, we suppose that C is a function of capital requirements as

$C(\alpha)$ and will explore the impact of varying these levels. In general, C could also change with v so that we seek a solution of

$$v = \hat{C}(\alpha, p, v)(I - C(\alpha, p, v))^{-1}(Dp - b(p, v). \tag{2.5}$$

To find this new best-case equilibrium, we will follow the same form of iterative solution but assuming that C is updated simultaneously with $b(p, v)$. As the examples below demonstrate, changing cross-holding levels in C in response to changes in capital requirements α can lead to substantially different sequences of failure cascades and resulting equilibria.

3 Optimization Model and its Inverse

The fundamental assumption in this model extension is that each organization (e.g, firm, bank, or regional collective) invests in a set of assets of both that institution and others. The remainder is retained as equity. We assume that the institution maximizes expected utility through its choice of cross-holdings and subject to any restrictions, such as capital requirements for a banking institution. The expected utility in this model is assumed to reflect single-period risk associated with continuous variation in returns (i.e., is ex post of discontinuous losses due to failures), although dynamic utility and discontinuous losses could also be included (e.g., building on the structure in Birge et al. (2016)).

For institution i representing a bank (or collection), the liabilities and assets are represented as

$$E_i + N_i = A_i - D_i, \tag{3.6}$$

where E_i is the equity of i, A_i are its assets, D_i are its deposits, and N_i are its non-core liabilities or short-term borrowing (i.e., liabilities consist of equity, deposits, and debt). We capture any capital requirements as constraints that imply an upper bound α_i on A_i relative to E_i. For the optimization, A_i is normalized to 1 so that $E_i \geq \alpha_i A_i = \alpha_i$.

We assume that each institution i has access to its own market of loans with expected returns r_i and with a covariance matrix Σ on these internal returns. Each institution is also assumed to have access to other institutions' market assets (e.g., either through direct lending in the region or through mutual ownership) but may face additional costs (or perhaps preferences) for investing in these other institutions (or regions). The model then attaches an unobserved (i.e., only privately known) transaction cost τ_{ij} to such loan investments of i in j. Institutions i pay depositors d_i, equity holders a dividend e_i, and can borrow short-term non-core funds at a rate n_i. They then solve an optimization problem to determine their asset allocation within their home market and in exchange with others. The overall allocation of assets from institution i is given as x_i.

While we can assume various forms of the objective in this model, the illustration here assumes that each institution maximizes an expected mean-variance utility with a given risk aversion parameter. This optimization model then takes the following form.

$$\max_{x_i,S_i,E_i} (r-\tau_i)^T x_i - \frac{\gamma_i}{2} x_i^T \Sigma x_i - d_i D_i - e_i E_i - n_i S_i \tag{3.7}$$

$$\text{s. t. } \mathbf{1}^T x = 1; \tag{3.8}$$

$$\mathbf{1}^T x - E_i - S_i = D_i; \tag{3.9}$$

$$E_i \geq \alpha_i; \tag{3.10}$$

$$x_i, S_i \geq 0, \tag{3.11}$$

where γ_i is i's risk aversion parameter and we assume that D_i is fixed in the short term and cannot be changed to increase loan amounts. For illustration, we also assume that the capital ratio constraint is binding so that $E_i = \alpha_i$ and then $S_i = 1 - \alpha_i - D_i$ (i.e., so that short-term borrowing scales according to the size of the loan portfolio and the capital ratio). With these assumptions and assuming deposits D_i are fixed, we can substitute $E_i = \alpha_i \mathbf{1}^T x_i$ and then write the optimality conditions for (3.7) as the following with $\mu_i \geq 0$:

$$(r-\tau_i) - (\gamma_i \Sigma + \alpha_i e_i \mathbf{1}^T) x_i + \mu_i = 0, \tag{3.12}$$

$$\mu_i^T x_i = 0, \tag{3.13}$$

where x can again be scaled so that $\mathbf{1}^T x_i = 1$.

Assuming that $x > 0$ (which, if not, can be adjusted to the positive quantities x_I), then (3.12) and feasibility in (3.7)–(3.11) reduces to:

$$(\gamma_i \Sigma + \alpha_i e_i \mathbf{1}^T) x_i \quad = \quad r - \tau_i; \tag{3.14}$$

which can be solved by noting $\tau_{ii} = 0$ for Σ_i as the ith row of Σ so that

$$(\gamma_i \Sigma_i + \alpha_i e_i \mathbf{1}^T) x_i = r_i,$$

which for $\mathbf{1}^T x_i = 1$ implies:

$$\gamma_i \Sigma_i x_i = r_i - \alpha_i e_i.$$

For the data used in the illustrative example below, we assume that Σ corresponds to historical observations of the returns for assets in different regions, r_i are the current returns, n_i are current short term rates in each region, α_i correspond to current leverage ratios, D_i are current deposits that are observable, and initial x_i's are also observed. Now, the estimation model can be set up to solve for γ_i and τ_i that will be consistent with these observations. If Σ_i and x_i are observed along with the rate r_i, capital ratio α_i and dividend rate e_i, then γ_i can be recovered as

$$\gamma_i = (r_i - \alpha_i e_i)/\Sigma_i x_i. \tag{3.15}$$

We can then solve for the effective transaction costs as:

$$\tau_{ij} = r_j - \alpha_i e_i - \gamma_i \Sigma_j x_i, \tag{3.16}$$

for each τ.

With these values, we can then use solutions of (3.7)-(3.11) to perform counterfactual analyses on the impact of changing capital ratio requirements and thereby generate updated values of C . The key observation is that changing the reserve requirements results in a different set of exposures as capital flows into other regions. The stability of the network then changes in response to these adjustments which should be considered in any analysis.

The overall procedure to find solutions of (2.5) is to follow the steps below following the identification of τ and γ using (3.15) and (3.16). The aggregate level of holdings of each organization i is given initially as h_i and the lower failure thresholds are given as $\bar{v}_i$ and losses given as $\beta_i(p)$. Each step of this method updates v until a solution of (2.5) is obtained.

1. For the current value of α, solve (3.7)–(3.11) for $x_i(\alpha)$ and let $X(\alpha) = [x_1(\alpha)x_2(\alpha)\dots x_n(\alpha)]$. Set $C_{ij}(\alpha) = \frac{x_{ij}h_i}{\sum_{i=1}^{n} x_{ij}h_i}$ for $i \neq j$ and $\hat{C}_{ii} = 1 - \sum_{j=1}^{n} C_{ji}$. Solve for $v = \hat{C}(I-C)^{-1}Dp$.

2. Let $\lambda_i = \beta_i(p)\mathbf{1}_{v_i < \bar{v}_i}$ and let $v' = \hat{C}(I-C)^{-1}(Dp - \lambda)$.

3. If $v' = v$, stop; otherwise, set $v = v'$ and return to Step 2.

This approach can also be adapted for matrices C that depend on v as well as α. This would be relevant if the expected returns, transactions costs, or anticipated volatilities represented in Σ were also functions of the organizational values (or their relative values).

4 Illustrative Example

As an illustrative example, we use similar data to that in Elliott et al. (2014) on cross-holdings of debt among nine countries (the Euro-based countries from Elliott et al. (2014) plus Japan, the United Kingdom, and the United States). The data on the total cross-holdings (in million $US) appear in Table 1 using the Q2 2015 report of the Bank of International Settlements (BIS, Table B4 in Bank of International Settlements (2015)). To compute the original matrix C_0 for current capital ratios α_0, we use the total debt amounts for each country from BIS as well. These values give $\hat{C}_0$, which is then combined to form the original dependency matrix $A_0 = \hat{C}_0(I - C_0)^{-1}$ that appears in Table 2.

As in Elliott et al. (2014), we assume that $D = i$ and p is proportional to the countries GDP (using Portugal's GDP as the base of 1). Using 2014 GDP from World Bank (2015)?, we then use the values for p in Table 3. From these original cross-holdings and primitive values, we can solve

	France	**Germany**	**Greece**	**Italy**	**Japan**	**Portugal**	**Spain**	**UK**	**USA**
France	–	158690	1330	285003	172268	13597	110840	234737	476762
Germany	186443	–	21040	97481	31165	15885	89294	415382	482631
Greece	1130	2047	–	347	52	33	101	11113	1797
Italy	43146	193754	805	–	5116	3232	43301	48056	32555
Japan	146676	89122	240	30469	–	539	20400	187862	1.31e06
Portugal	4435	2078	185	6242	242	–	12963	2758	2387
Spain	43548	89294	1001	46750	9510	63082	–	422592	257367
UK	172019	126033	4912	32537	98412	10512	28169	–	917815
USA	193151	152835	1429	60734	314551	5345	40910	477759	–

Table 1: Consolidated positions in millions of US dollars of counterparties from the row entry countries on counterparties resident in the column entry from the Bank on International Settlements Q2 2015 report (Bank of International Settlements (2015)).

	France	**Germany**	**Greece**	**Italy**	**Japan**	**Portugal**	**Spain**	**UK**	**USA**
France	0.82	0.03	0.01	0.07	0.01	0.04	0.04	0.05	0.04
Germany	0.04	0.86	0.06	0.03	0.00	0.05	0.04	0.09	0.04
Greece	0.00	0.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00
Italy	0.01	0.03	0.00	0.83	0.00	0.01	0.02	0.01	0.00
Japan	0.04	0.02	0.00	0.02	0.95	0.01	0.01	0.06	0.10
Portugal	0.00	0.00	0.00	0.00	0.00	0.69	0.00	0.00	0.00
Spain	0.01	0.02	0.01	0.02	0.00	0.15	0.85	0.09	0.02
UK	0.03	0.02	0.01	0.01	0.01	0.02	0.01	0.61	0.04
USA	0.04	0.02	0.01	0.02	0.02	0.02	0.02	0.09	0.75

Table 2: Original dependency matrix $A_0 = \hat{C}_0(I - C_0)^{-1}$.

Country	Relative GDP
France	12.29
Germany	16.81
Greece	1.02
Italy	9.30
Japan	20.00
Portugal	1.00
Spain	6.00
United Kingdom	12.99
United States	75.70

Table 3: Original primitive asset values p.

for an original market value $v_0 = A_0 p$ as:

$$v_0 = \begin{pmatrix} 15.2838 \\ 19.9137 \\ 0.9863 \\ 9.0642 \\ 28.3350 \\ 0.7829 \\ 8.8020 \\ 12.1361 \\ 59.8130 \end{pmatrix}. \tag{4.17}$$

The next steps require estimates of the parameters of the optimization model in (3.7)–(3.11). From World Bank (2015), we obtain the original capital ratio α_0. From Trading Economics (2015), we obtain the base bank lending rates r for 2015 and use annual rates from 1997 to 2015 used to calculate the covariance matrix Σ. For the assumed dividend rates e, we use 1% plus the deposit rates as averages of those given in Deposits.org (2015). We then calculate the implied risk-aversion coefficients γ from (3.15) and the implied transaction costs τ from (3.16). The parameter values used here for α_0, r, e, and implied γ appear in Table 4. While these results are purely for illustration here, they suggest high risk tolerance in Greece and low risk tolerance in Japan. i The implied transaction costs appear in Table 5. Again, these results are only for illustration but suggest

Country	Cap Ratio α_0	Rate r	Dividend e	Risk Coeff. γ
France	5.30	1.82	2.00	0.70
Germany	5.80	2.64	1.40	0.94
Greece	7.70	5.51	2.80	0.14
Italy	5.90	2.83	2.20	0.42
Japan	5.80	1.15	1.20	1.51
Portugal	8.00	3.99	1.60	0.55
Spain	7.40	2.68	1.30	0.54
UK	5.60	1.50	2.00	0.45
USA	11.60	3.25	2.00	1.35

Table 4: Initial capital ratios, lending rates, assumed dividend rates (all in %), and implied risk coefficient γ values.

Country	France	Germany	Greece	Italy	Japan	Portugal	Spain	UK	USA
France	0.00	0.74	2.40	0.18	0.25	2.27	0.33	-0.56	1.62
Germany	-0.64	0.00	2.41	-0.50	0.00	2.17	-0.25	-1.33	0.95
Greece	1.08	2.04	0.00	0.97	0.55	1.61	0.92	0.78	2.82
Italy	0.26	1.12	0.51	0.00	0.29	1.56	0.13	-0.22	2.02
Japan	-0.13	0.61	1.22	-0.19	0.00	1.96	0.02	-0.74	1.49
Portugal	0.47	1.51	-3.22	-0.46	0.31	0.00	-0.40	0.20	2.49
Spain	0.16	1.00	0.33	-0.14	0.27	1.48	0.00	-0.35	1.90
UK	0.56	1.29	3.86	1.10	0.51	3.04	1.16	0.00	2.03
USA	-1.32	-0.80	2.84	-1.09	-0.38	2.22	-0.75	-2.23	0.00

Table 5: Implied transaction costs τ (in %) for lending from the row entry to the column entry country.

differences in relative preferences and costs for specific cross-holdings.

To see the effects of failure cascades in this regime, we repeat the exercise from Elliott et al. (2014) of setting a threshold level θ as a fraction of 2008 GDP levels and setting losses given failure at $\beta_i = 0.5$ for each i. We first observe the sequence of failures for the base case of the capital ratios α_0 in Table 6. The critical threshold values θ give the lowest failure threshold for the given set of countries to experience a failure at any of the iterations of Step 2 (or hierarchy levels) of the algorithm in Section 3 to determine best case equilibria. At a threshold of $\theta = 1.01$, for example, Germany, Greece, and Italy fail for the first iteration, which then yields of a failure for Japan on the next iteration, and a failure for Portugal on the third iteration. Comparing to the data for the six Euro-based countries used in Elliott et al. (2014), the difference in the sequence of failures here

	Critical Threshold Values θ								
Levels:	**0.68**	**0.89**	**0.91**	**0.92**	**0.96**	**0.99**	**1.00**	**1.04**	**1.14**
First	GR	GR	GR, PT	GR, IT, PT	GR, IT, PT, ES	GR, IT, PT, ES	FR, GR, IT, PT ES	FR, GR, IT, JP, PT, ES	All except US
Second	-	PT	IT	ES	FR	FR	JP	DE	US
Third	-	-	-	-	-	JP	DE	GB	-

Table 6: Failure cascades for increasing failure threshold values θ with the level of the failures for the base case of original capital ratios α_0 for France (FR), Germany (DE), Greece (GR), Italy (IT), Japan (JP), Portugal (PT), Spain (ES), United Kingdom (GB), and United States (US).

is that Italy fails next after Portugal, instead of being the last to fail in the example from Elliott et al. (2014) (which may be a result of Italy's reduced market value relative to prior GDP).

To see the effects of increasing capital ratios, we update the capital ratios to α_1 defined by $\alpha_1(i) = \max(0.06, \alpha_0(i), i = 1, \ldots, 9$. This then requires recalculating the dependency matrix A at A_1 using new values $\hat{C}_1$ and C_1 as function of new holdings x_1. With this new matrix, we then repeat the exercise of increasing the failure threshold θ and observing the cascade of failures. The results appear in Table 7. In this case, Italy, Germany, and Japan all fail before Portugal, a result of both the vulnerability of Italy as in the base case, and the tighter capital requirements leading German and Japanese investment to shift to Italy and Germany, respectively, which then experience failures.

Another observations from the case of capital ratios increased to α_1 (a minimum of 6%) is that the threshold value that leads all organizations (including the US) to fail is at $\theta = 1.142$, which is lower than the corresponding value of $\theta = 1.144$ for the base case. By increasing capital requirements then, not only has the critical value for the first failures increased, but the critical threshold for failures in all organizations is increased as well.

We perform the exercise for even tighter restrictions in Table 8. In this case, the shift in investments follows a revision of the capital ratios to α_2 defined by $\alpha_2(i) = \max(0.08, \alpha_0(i), i = 1, \ldots, 9$. Again, we obtain a new dependency matrix A as A_2 using new values $\hat{C}_2$ and C_2 from new investment levels x_2. The table shows the results of increasing the threshold values as before. Now, Italy is first to fail because the high capital ratio has caused the Italian institutions to increase holdings in Greece (whose value decline then causes Italy's value to collapse). Germany strongly

	Critical Threshold Values θ								
Levels:	**0.65**	**0.74**	**0.80**	**1.00**	**1.01**	**1.04**	**1.08**	**1.11**	**1.14**
First	GR	GR	GR, IT	DE, GR, IT	DE, GR, IT	DE, GR, IT, JP, PT	DE, GR, IT, JP, PT, ES	DE, GR, IT, JP, PT, ES	DE, GR, IT, JP, PT, ES
Second	-	IT	DE	JP	JP	ES	FR	FR	FR, GB
Third	-	-	-	-	PT	-	-	GB	US

Table 7: Failure cascades for increasing failure threshold values θ with the level of the failures for the case of capital ratios $\alpha_1(i) = \max(\alpha_0(i), 0.06), i = 1, \ldots, 9$, for France (FR), Germany (DE), Greece (GR), Italy (IT), Japan (JP), Portugal (PT), Spain (ES), United Kingdom (GB), and United States (US).

	Critical Threshold Values θ							
Levels:	**0.59**	**0.60**	**0.68**	**0.92**	**0.99**	**1.04**	**1.17**	**1.21**
First	IT	IT	DE, IT	DE, GR, IT	DE, GR, IT	DE, GR, IT	FR, DE, GR, IT, JP, ES	FR, DE, GR, IT, JP, ES
Second	-	DE	GR	FR	FR	FR, ES	PT	PT
Third	-	-	-	-	ES	JP	-	US
Fourth	-	-	-	-	-	-	-	GB

Table 8: Failure cascades for increasing failure threshold values θ with the level of the failures for the case of capital ratios $\alpha_2(i) = \max(\alpha_0(i), 0.08), i = 1, \ldots, 9$, for France (FR), Germany (DE), Greece (GR), Italy (IT), Japan (JP), Portugal (PT), Spain (ES), United Kingdom (GB), and United States (US).

increases investment in Italy leading to its early failure. France becomes an early casualty in this scenario as well since the tighter capital requirements lead to greater cross-holding in Germany. Portugal, whose capital ratio was not affected, remains balanced and fails at a later point in this scenario compared to the other cases.

These results are again meant only to be illustrative examples of the differences that can occur as organization react to changes in capital requirements. They show how institutions' perceptions of the value of cross-holdings and their risk preferences can affect the overall level of risk in the system and that even minor changes may have relatively large effects on the fragility of the system and the propagation of risk.

5 Conclusions and Extensions

This paper has presented a model of financial networks in which the constituent institutions endogenously determine their cross-holdings in reaction to perceptions about the relative merits of investments in the other parties and in requirements on the level of equity. The basic assumption is that the organizations maximize an objective given these perceptions and requirements, which then results in observed levels of cross-holdings as in the debt from one country held by others. Assuming that these implied costs and benefits from cross-investment remains constant then allows for counter-factual considerations of the risk in the overall system. The paper then illustrates how this can be used to simulate cascades of failures under varying capital requirements.

The validity of such a modeling exercise could be tested with more thorough empirical means given data on cross-holdings and capital requirements. In particular, such empirical studies would benefit from observations of exogenous changes in capital requirements and the subsequent changes in cross-holdings. Finding such examples and isolating these effects would present a challenge but might be overcome with sufficiently detailed data.

In addition to general enhancements of the network model such as failure-contingent changes in cross-holdings, many of the assumptions in this paper can be relaxed or altered in extensions of the modeling framework. As noted, the form of the institutions' objective could be enhanced to include more dynamics and anticipation of failures and of their losses. Such enhancements of course present additional challenges for calibration, but could provide more robust results for counter-factual analyses.

More detailed models of cross-holdings and agency issues could also be included into the modeling framework. With additional empirical support and refinement of the agents' choice problems, the model could then be used to help in assessing changes to policies such as capital requirements or conditions for investments in different markets. In addition, the model could be useful in assessing perceptions about the relative value of alternative investments and costs. Modifications, for example through mergers and acquisitions, and their affect on transaction costs could then be assessed in this context as well.

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